# DO NOW

Integrate: $\int x^2(x^3 - 1)^4 dx$

$\int u^4 \cdot \frac{1}{3} du$

$\frac{1}{3}\int u^4 du$

$\frac{1}{3} \cdot \frac{u^5}{5} + C$

$\boxed{\frac{(x^3-1)^5}{15} + C}$

$u = x^3 - 1$
$du = 3x^2 dx$
$\frac{1}{3} du = x^2 dx$



## 5.6 Numerical Integration

Used to evaluate a definite integral involving a function whose antiderivative cannot be found.

Approximation techniques:

1. Trapezoidal Rule
   ↳ trapezoids $A = \frac{1}{2}h(b_1 + b_2)$
2. Simpson's Rule
   ↳ parabolas...



**Trapezoidal Rule:**

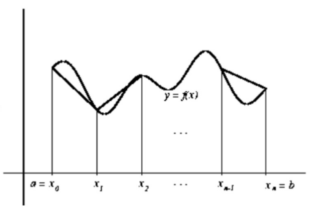
Let $f$ be continuous on $[a, b]$.

$$\int_a^b f(x)\,dx = \frac{1}{2}\Delta x\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots . 2f(x_{n-1}) + f(x_n)\right]$$

where $\Delta x = \frac{b-a}{n}$

and $x_k = a + k\Delta x$

***The larger the value of n, the better the approximation.






Example: Use the trapezoidal rule to approximate:

$\int_{-1}^{2} x^2 dx;\ n = 4$ $\Delta x = \frac{2 - -1}{4} = \frac{3}{4}$

$x_0 = -1$
$x_1 = -\frac{1}{4}$
$x_2 = \frac{1}{2}$
$x_3 = \frac{5}{4}$
$x_4 = 2$

$\frac{1}{2}\Delta x\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$

$\frac{1}{2} \cdot \frac{3}{4}\left[f(-1) + 2f(-\frac{1}{4}) + 2f(\frac{1}{2}) + 2f(\frac{5}{4}) + f(2)\right]$

$\frac{3}{8}\left[1 + 2 \cdot \frac{1}{16} + 2 \cdot \frac{1}{4} + 2 \cdot \frac{25}{16} + 4\right]$

$\frac{3}{8}\left[5 + \frac{1}{8} + \frac{1}{2} + \frac{25}{8}\right]$

$\frac{3}{8}\left[5.5 + \frac{26}{8}\right]$

$\frac{3}{8}\left[8.75\right]$

$\boxed{3.2813}$

Actual:

$\int_{-1}^{2} x^2 dx = \left[\frac{1}{3}x^3\right]_{-1}^{2}$

$\frac{1}{3}(8) - \frac{1}{3}(-1)$

$\frac{8}{3} + \frac{1}{3}$

$\boxed{3}$

Error: $3 - 3.2813$
↳ $\boxed{-.2813}$



**Simpson's Rule:**

Let $f$ be continuous on $[a, b]$.

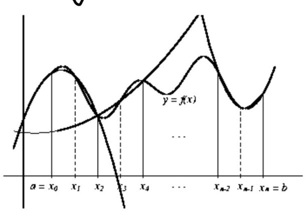
$$\int_a^b f(x)\,dx = \frac{1}{3}\Delta x\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots . + 4f(x_{n-1}) + f(x_n)\right]$$

where $\Delta x = \frac{b-a}{n}$

and $x_k = a + k\Delta x$

and $n$ is an even integer

***The larger the value of n, the better the approximation.






Example: Use Simpson's Rule to approximate:

$\int_1^2 \frac{1}{x} dx;\ n = 10$ $\Delta x = \frac{2-1}{10} = .1$

$x_0 = 1$
$x_1 = 1.1$
$x_2 = 1.2$
$x_3 = 1.3$
$\vdots$
$x_{10} = 2$

$\frac{1}{3}\Delta x\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 4f(x_9) + f(x_{10})\right]$

$\frac{1}{3} \cdot \frac{1}{10}\left[\frac{1}{1} + 4 \cdot \frac{1}{1.1} + 2 \cdot \frac{1}{1.2} + 4 \cdot \frac{1}{1.3} + \frac{2}{1.4} + \frac{4}{1.5} + \frac{2}{1.6} + \frac{4}{1.7} + \frac{2}{1.8} + \frac{4}{1.9} + \frac{1}{2}\right]$

$\frac{1}{30}\left[20.7945\right]$

$\boxed{.6932}$

Actual:

$\int_1^2 \frac{1}{x} dx = \left[\ln|x|\right]_1^2$

$\ln 2 - \ln 1$

$\ln 2$

$\boxed{.6931}$

Error: $.6931 - .6932$

$\boxed{-.0001}$

There are formulas for computing the error in Trapezoidal and Simpson's Rules.

If interested, see page 349.



# HOMEWORK

pg 350; 1, 5, 9, 11